Instructions: Try these problems from the textbook as practice (answers below).

Problems

9-11. Evaluate the double integral by first identifying it as the volume of a solid.

9. $\iint_R \sqrt{2}\ dA,\ R = \{(x,y) : 2 \le x \le 6, -1 \le y \le 5\}$

11. $\iint_R (4-2y)\ dA,\ R = [0,1] \times [0,1]$

13. Find $\int_0^2 f(x,y)\ dx$ and $\int_0^3 f(x,y)\ dy$ where $f(x,y) = x + 3x^2y^2$.

15-26. Calculate the interated integral.

15. $\int_1^4 \int_0^2 (6x^2y - 2x)\ dy\ dx$

17. $\int_0^1 \int_1^2 (x + e^{-y})\ dx\ dy$

21. $\int_{-3}^3 \int_0^{\pi/2} (y + y^2 \cos x)\ dx\ dy$

23. $\int_0^3 \int_0^{\pi/2} t^2 \sin^3(\phi)\ d\phi\ dt$

25. $\int_0^1 \int_0^1 v(u+v^2)^4\ du\ dv$

27-34. Calculate the double integral.

27. $\iint_R x \sec^2 y\ dA,\ R = [0,2] \times [0, \pi/4]$

29. $\iint_R \dfrac{xy^2}{x^2+1}\ dA,\ R = [0,1] \times [-3,3]$

31. $\iint_R x \sin(x+y)\ dA,\ R = [0,\pi/6] \times [0,\pi/3]$

33. $\iint_R ye^{-xy}\ dA,\ R = [0,2] \times [0,3]$

35-37. Sketch the solid whose volume is given by the interated integral.

35. $\int_0^1 \int_0^1 (4 - x - 2y)\ dx\ dy$

37. $\int_{-2}^2 \int_{-1}^3 (4 - x^2)\ dx\ dy$

43. Find the volume of the solid that lies under the plane $4x + 6y - 2z + 15 = 0$ and above the rectangle $R = [-1,2] \times [-1,1]$.

45. Find the volume of the solid lying under the elliptic paraboloid $x^2/4 + y^2/9 + z = 1$ and above the rectangle $R = [-1,1] \times [-2,2]$.

47. Find the volume of the solid enclosed but the surface $z = 1 + x^y e^y$ and the planes $z = 0,\ x = \pm 1,\ y = 0$ and $y = 1$.

49. Find the volume of the solid enclosed by the paraboloid $z = 2 + x^2 + (y-2)^2$ and the planes $z = 1$, $x = 1$, $x = -1$, $y = 0$, and $y = 4$.

53. Find the average value of $f(x,y) = x^2y$ over the rectangle R which has vertices $(-1,0)$, $(-1,5)$, $(1,5)$, $(1,0)$.

55. Use symmetry to evaluate $\iint_R \dfrac{xy}{1+x^4}\ dA$ where $R = [-1,1] \times [0,1]$.

Answers

9. $24\sqrt{2}$
11. 3
13. $2 + 8y^2$, $3x + 27x^2$
15. 222
17. $\frac{5}{2} - e^{-1}$
21. 18
23. 6
25. $\frac{31}{30}$
27. 2
29. $9 \ln 2$
31. $\frac{1}{2}(\sqrt{3} - 1) - \frac{1}{12}\pi$
33. $\frac{1}{2}e^{-6} + \frac{5}{2}$
43. 51
45. $\frac{166}{27}$
47. $\frac{8}{3}$
49. $\frac{64}{3}$
53. $\frac{5}{6}$
55. 0